Minitest 1, Linear Algebra

Dr. Adam Graham-Squire, Fall 2017

Name: Key

I pledge that I have neither given nor received any unauthorized assistance on this exam.

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(*signature*)

DIRECTIONS

1. Don't panic.
2. Show/explain all of your work. A correct answer with insufficient work will lose points.
3. Read each question carefully, and make sure you answer the the question that is asked. If the question asks for an explanation, make sure you give one.
4. Clearly indicate your answer by putting a box around it.
5. Calculators are allowed on the all questions of the exam, however you still must show your work even if you confirm it with a calculator. For the last question, you can use a computer if you want.
6. Make sure you sign the pledge.
7. Number of questions = 5. Total Points = 25.

1. (5 points) For the system of equations

$$\begin{aligned} x_1 + x_2 - x_3 &= 5 \\ 2x_2 + 4x_3 &= 11 \\ 2x_1 + 4x_2 + 2x_3 &= 21 \\ x_1 - 2x_2 - 7x_3 &= 12 \end{aligned}$$

do the following:

0.5 (a) Write the system in matrix-vector form.

3.5 (b) Write the augmented matrix, then row reduce the matrix to reduced row echelon form.

1 (c) Find all solutions and write your answer in parametric vector form (if solutions exist).

Note: You can use a calculator to check your work, but you must show the steps of the row reduction to receive full credit.

(a) $\begin{bmatrix} 1 & 1 & -1 \\ 0 & 2 & 4 \\ 2 & 4 & 2 \\ 1 & -2 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 11 \\ 21 \\ 12 \end{bmatrix}$ ✓

(b) $\left[\begin{array}{ccc|c} 1 & 1 & -1 & 5 \\ 0 & 2 & 4 & 11 \\ 2 & 4 & 2 & 21 \\ 1 & -2 & -7 & 12 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & -1 & 5 \\ 0 & 1 & 2 & 5.5 \\ 0 & 2 & 4 & 11 \\ 0 & -3 & -6 & 7 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & -1 & 5 \\ 0 & 1 & 2 & 5.5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 23.5 \end{array}\right]$

$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -3 & -0.5 \\ 0 & 1 & 2 & 5.5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 23.5 \end{array}\right]$ ✓

inconsistent! ✓

(c) No solutions exist, so cannot put in param. vector form

2. (5 points) Determine all values of h and k such that the system of equations

$$\begin{aligned} 3x_1 + 6x_2 &= h \\ 5x_1 + kx_2 &= 10 \end{aligned}$$

has

(a) No solution.

(b) Infinitely many solutions.

(c) A unique solution.

$$\left[\begin{array}{cc|c} 3 & 6 & h \\ 5 & k & 10 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & 2 & h/3 \\ 0 & k-10 & -\frac{5h}{3}+10 \end{array}\right] \checkmark$$

1.5 (a) if $k-10=0$ and $\frac{-5h}{3}+10 \neq 0$, then system is inconsistent $\Rightarrow$ $\boxed{k=10 \text{ and } h \neq 6}$

1.5 (b) if $k-10=0$ and $\frac{-5h}{3}+10=0$ then x_2 is free $\Rightarrow$ ∞ solutions $\Rightarrow$ $\boxed{k=10 \text{ and } h=6}$

✓ (c) if $k-10 \neq 0$, then $k-10$ spot is a pivot and get unique solution $\Rightarrow$ $\boxed{k \neq 10}$ h can be whatever.

3. (5 points) For each matrix A, answer the following questions. Make sure you (briefly) explain your answer.

(i) Does the homogeneous equation $A\mathbf{x} = \mathbf{0}$ have a nontrivial solution?

(ii) Does the equation $A\mathbf{x} = \mathbf{b}$ have at least one solution for every possible $\mathbf{b}$?

(a) A is a $\overset{2}{\cancel{4}} \times \overset{3}{\cancel{2}}$ matrix with 2 pivots.

2.5

~~(i) No nontrivial solutions b/c no free variables~~

~~(ii) $Ax=b$ does not have a solution for all b b/c not a pivot in each row.~~

$$\begin{array}{ccc} & & \text{free} \\ & & \downarrow \\ \end{array}$$
$$\begin{bmatrix} * & * & * \\ 0 & * & * \end{bmatrix} \checkmark$$

(i) Yes, b/c has free variables! ✓

(ii) Yes, b/c pivot in every row ✓

(b) A is a 5×5 matrix with $\overset{3}{\cancel{3}}$ pivots.

2.5

(i) has 2 free variables

$\Rightarrow$ Yes ✓ nontrivial solutions! ✓

(ii) No, ✓ b/c not a pivot ✓ in each row.

4. (5 points) Let A be a 2×3 matrix, $\mathbf{v}_1$ and $\mathbf{v}_2$ be vectors in $\mathbb{R}^2$, and let $\mathbf{w} = \mathbf{v}_1 + \mathbf{v}_2$. Now suppose that $A\mathbf{u}_1 = \mathbf{v}_1$ and $A\mathbf{u}_2 = \mathbf{v}_2$ for some vectors $\mathbf{u}_1$ and $\mathbf{u}_2$ in $\mathbb{R}^3$. Explain why the system $A\mathbf{x} = \mathbf{w}$ is consistent. (Hint: you can actually state what the solution to the equation will be).

$Au_1 + Au_2 = v_1 + v_2 = w$ ✓✓

$\Rightarrow A(u_1 + u_2) = w$ ✓

So for $\vec{x} = u_1 + u_2$, $Ax = w$ and thus ✓

$Ax = w$ has a solution so it is ✓

consistent!

Extra Credit(1 point) For each question, state if it is True or False. If True, briefly justify why it is true. If False, give an explanation or a counterexample to show why it is false.

(a) If an augmented matrix represents a system that has an infinite number of solutions, then it must have a bottom row of all zeroes .

False!

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 4 \\ 0 & 1 & 2 & 5 \end{array}\right]$$

$\uparrow$ free variable

This is a consistent matrix with a free variable $\Rightarrow$ infinite # solutions

but <u>no</u> row of zeroes

(b) The equation $A\mathbf{x} = \mathbf{b}$ is homogeneous if the zero vector is a solution.

True! If $A\vec{0} = b$, then $\vec{0} = b$ b/c $A\vec{0} = \vec{0}$, and $Ax = \vec{0}$ is a homogeneous equation.

Technology Question:

5. (5 points) For the given matrix A, use some form of technology (graphing calculator, online matrix calculator, Maple, etc) to answer the questions below. You do not need to show work of solving the matrix (since that will be done by the technology) but you should explain what you did and what your results are.

$$\begin{bmatrix} 12 & 10 & -6 & -3 & 7 \\ -7 & -6 & 4 & 7 & -9 \\ 9 & 9 & -9 & -5 & 5 \\ 31 & 27 & -19 & 0 & 8 \end{bmatrix} \xrightarrow{\text{computer reduces to}} \begin{bmatrix} 1 & 0 & 2 & 0 & 2 \\ 0 & 1 & -3 & 0 & -2 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) Do the columns of the matrix span $\mathbb{R}^4$? Explain why or why not.

(b) Suppose the columns of A are denoted $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4, \mathbf{v}_5$. Then Span$\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4, \mathbf{v}_5\}$ is the subspace of $\mathbb{R}^4$ (possibly all of $\mathbb{R}^4$) spanned by $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ and $\mathbf{v}_5$. Can you remove any of the columns $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ or $\mathbf{v}_5$ and not change the Span? If so, which column vectors and why? If not, why not?

(a) No, b/c there is not a pivot in every row! any $\vec{b} \in \mathbb{R}^4$ that reduces to have a nonzero entry in bottom position will be inconsistent

(b) Yes! You can remove any of the vectors! x_3 and x_5 are free, so removing them make no diffence.

If you ~~you~~ removed x_1 or x_2, then x_3 would become a pivot, and if you removed x_4 then x_5 becomes a pivot.

$\boxed{-0.5 \text{ if only mention } v_3 \text{ and } v_5}$

$\boxed{-1 \text{ if only } v_3 \text{ or } v_5 \text{ and nothing else}}$